Energy trapping in extensional thin film MEMS resonators and applications to filtering at UHF frequencies.

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Abstract—**In this paper we apply the energy trapping theory to the extensional modes of 6mm materials and particularly to piezoelectric films textured in a direction normal to the film. The 6mm symmetries imply that the Mn' and Pn' coefficients of the Tiersten equation are equal and that energy trapping can nearly always be obtained (most often using "inverse" energy trapping with Mn'=Pn'<0). In this case, particular dispositions have to be taken to obtain energy trapping (in inverting of the order of the cut off frequencies of the different regions as compared to conventional energy trapping). The 6mm symmetries imply also that resonators geometries having a revolution axis lead to more interesting properties in term of spectral purity and of electric properties (optimal character that can be demonstrated). Example of designs of filter type AlN resonators using round or annular electrodes are presented. It is observed that a wide variety of equivalent inductance can be obtained particularly in using a very interesting properties of the annular electrodes. Application of these resonators to UHF filtering is then discussed, it is observed that interesting responses and low termination impedances can be obtained using symmetrical full lattice filters "enlarged" with serial or parallel inductances.** *(Abstract)*

I. *INTRODUCTION "INVERSE" ENERGY TRAPPING*

Miniaturized resonators using the extensional modes generated in textured thin films of aluminum nitride or zinc oxide are becoming widely used to constitute filters for UHF radio-communication terminals. However, in the present state of the art, large resonators using several forms of lateral confinement less efficient than "true" energy trapping are used. Several authors [1] [2], have some time ago, shown the possibility to make resonators using thickness modes confined near the centre of the device using the so call "backward" (or "inverse" or "reverse") energy trapping phenomena. This trapping effect can be obtained provided several conditions are fulfilled concerning the energy trapping parameters and the device design. In this paper, we will observe that these conditions can be obtained for the thickness extensional modes of the 6mm (∞mm) thin textured films, propagating in the 6 (∞) fold axis direction and we will make numerical investigations of the properties of such devices.

II. *SIMPLE THEORY OF REVERSE ENERGY TRAPPING FOR EXTENSIONAL MODES IN TEXTURED THIN FILM DEVICES.*

We use the formalism and the notations of Tiersten and Stevens [3] and those of references [4] and [6]. We consider the case of self supported thin piezoelectric films with deposited very thin metals and dielectrics films (and/or with zones of slightly lower thickness (grooved)). The lateral dependence of the main component of the vibration modes in plane energy trapping devices are governed by the Tiersten equations [3] valid for the different regions of the resonators. They have the form of bidimensional Helmotz equations with a forcing term (for the region of the electrodes) and concern, in fact, the lateral dependence of the main component of a transformed displacement (**û**). This transformed displacement appears in fact in a transformation made to replace the inhomogeneous (forcing) boundary condition at the surface of the electroded region by a forcing term in the corresponding equation. For 6mm films having the thickness in the direction of the 6 fold axis, the coefficients of the second order partial derivatives (Mn' and Pn') appearing in these equations should be equal (transverse isotropy of the films textured in a direction normal to their surface) and the Tiersten equations can be transformed into separable ones (1) in a particular polar coordinates (r,t) system deriving simply from the usual polar coordinates (ρ,θ) [6].

$$r=\sqrt{\frac{\alpha.C^*}{Mn}}.\gamma \ , \ t=\theta \quad \text{with} \quad \gamma_4=\frac{n^2\pi^2}{2h^4}$$

$$\tilde{u}^n_{1,r,r}+\frac{1}{r}\tilde{u}^n_{1,r}+\frac{1}{r^2}\tilde{u}^n_{1,t,t}+.A^*\tilde{u}^n_1=G(V) \quad (1)$$

$$A^* = \frac{\alpha . n . \pi}{4} \left| \frac{f_{n,m,p}^2 - f^{*2}}{f^{*2}} \right|$$

In this equation n is the overtone rank, f* is the cut-off frequency of the considered region. The separation of this 2nd order partial derivative equation leads to a Bessel or modified Bessel equation depending on the sign of A* and to an equation which has for solution the sine and cosine of mt, where m is a separation constant that has to be an integer to respect the periodicity in mt. The symmetric solutions (in x_1 and x_3) of the homogeneous equations (at V=0) have thus the form:

$$\tilde{u}_1^{nm} = \left\{ X_m^n . Be_m^1(k^* r) + Y_m^n . Be_m^2(k^* . r) \right\} \cos(mt)$$

where $k^* = \sqrt{|A^*|}$ and m integer

Where the Be_m functions are 1st and 2nd kind Bessel or modified Bessel functions of order m, depending on the sign of A*. To have a form of energy trapping the displacement should decrease rapidly in the outer region (here supposed infinite to simplify) and so to be the functions of imaginary argument K_m. The α parameter is the common sign of Mn and Pn. (α=+1 corresponds to the conventional energy trapping, α=-1 to inverse energy trapping). For the extensional modes of AlN and ZnO textured films α=-1, so to have energy trapping A* must then be positive in the central electroded region and negative outside. The cut off frequency of the outer region (f_{cl}) must then be lower than that of the central region (f_{ce}) (the contrary of what is made to have conventional energy trapping, when Mn' and Pn' are >0).

$$f_{cl} < f_{n,m,p}^r < f_{ce}$$

This can be obtained using simple dispositions like those represented on figure 1. To simplify (figures 1 and 2), we consider 2 mass loading, one for the external region (R'') one for the electrode region (R') whose difference is (approximately) an effective mass loading (R, negative). As shown in reference [1], it is possible to extend this kind of theory, to more intricate structures involving multiple (and thicker) layers provided they use materials with certain symmetries.

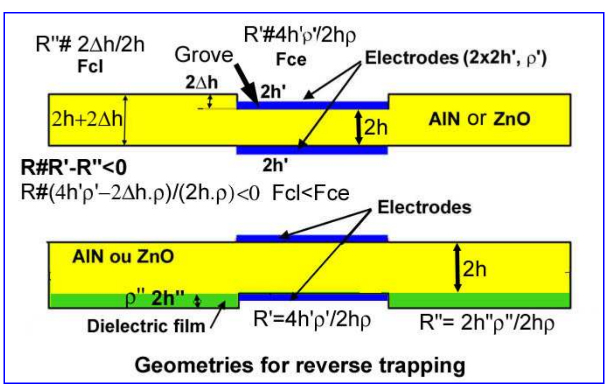

Figure 1: Obtainment of inverse energy trapping (when α=-1)

When the resonator uses round electrodes (figure 2) (and have a sufficiently large external radius so that the external region can be considered as infinite, to simplify), the eigen solution are very simples in the two regions:

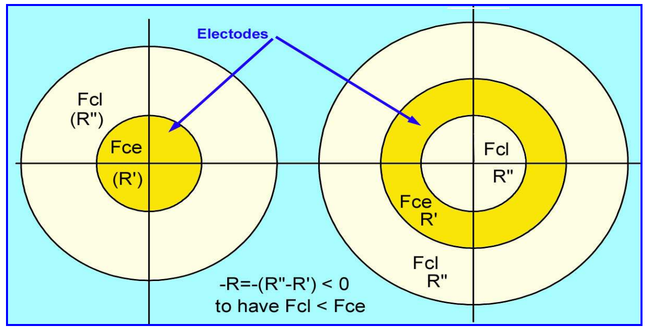

Figure 2: Resonators with revolution symmetry

$$\begin{cases} \hat{u}_{1e}^{nm} = B_m^n J_m(\hat{k}^n . r) \cos mt \sin(n\pi x_2 / 2h) \\ \hat{u}_{1l}^{nm} = C_m^n K_m(\bar{k}^n . r) \cos mt \sin(n\pi x_2 / 2h) \end{cases}$$

with $\hat{u}_1 = \tilde{u}_1(x_1, x_3).\cos(n\pi x_2 / 2h)$

For the electroded regions we have:

$$\hat{k}_n = \sqrt{\frac{\alpha . n . \pi \left(f_{ce}^2 - f_{n,m,p}^2 \right)}{4 . f_{ce}^2}} = \sqrt{|\hat{A}|}$$

$$fce = \frac{n}{2.2h} \sqrt{\frac{\bar{C}}{\rho}} \left[1 - 2R' - \frac{8k^2}{n^2 \pi^2} \right]$$

$\bar{C} = C_{33}(1 + k^2)$ with $k^2 = k_{33}^2$

For the unelectroded regions we have:

$$\bar{k}_n = \sqrt{\frac{\alpha . n . \pi \left(f_{n,m,p}^2 - f_{cl}^2 \right)}{4 . f_{cl}^2}} = \sqrt{|A|}$$

$$fcl = \frac{n}{2.2h} \sqrt{\frac{\bar{C}}{\rho}} \left[1 - 2R'' \right]$$

The index m appearing is an integer separation constant, m is even starting from 0 for the symmetric modes (symmetric in x_1 and x_3). With plain electrodes only the modes with m=0 can be excited, due to the symmetries existing in the mode shape for m>0 that cancels the electrical charge (current) induced on the electrodes by the exciting voltage. However the excitation of the modes with m≠0 is possible using electrodes divided in 2m regular sectors (see reference [5]). The transformed displacement (**û**), is identical to the usual displacement for the short circuits eigen modes but not for the forced modes [1,3,4,6].

If we consider now annular electrodes (figure 2), we have 3 regions one for the electrodes (region II, characterized with a cut off frequency fce and two unelectroded regions that are respectively the central one (I) and the most external one (III considered hereafter as infinite to simplify). Here we suppose also that the unelectroded regions (I and III) have the same cut-off frequency (fcl). The three homogeneous equations governing the eigen mode at V=0, in the three regions have then exact solutions of the form:

$$\begin{cases} \hat{u}_{1I}^{nm} = A_m^n I_m(\bar{k}^n.r)\cos mt.\sin(n\pi x_2/2h) \\ \hat{u}_{1II}^{nm} = \left\{B_m^n J_m(\hat{k}^n.r) + C_m^n Y_m(\hat{k}^n.r)\right\}\cos mt.\sin(n\pi x_2/2h) \\ \hat{u}_{1III}^{nm} = D_m^n K_m(\bar{k}^n.r)\cos mt.\sin(n\pi x_2/2h) \end{cases}$$

In both cases, these eigen solutions have to verify continuity conditions (continuity of the displacement and of its normal derivative) at the boundary between the regions and (eventually) a boundary conditions at the outer limit of the external region (in this case the solution for the external region (II or III) includes a supplementary term of the form $E_m.\ I_m(k_n.r)$). These conditions need to be expressed for each value of m and n and for 1, 2 or 3 values of the variable r corresponding to theses boundaries. An homogeneous system in the coefficients (A, B, C, ….) appearing in the expression of the displacement in the different regions is thus obtained for each values of n and m. The determinants of these systems must vanish to have non trivial solutions. This leads to equations verified by the resonance frequencies. Theses equations are solved numerically so that the eigen (resonance) frequencies $f_{n,m,p}$ are found. Then for each resonance frequency ($f_{n,m,p}$), the A, B, C,…coefficients are calculated as a function of one of them in solving the corresponding homogeneous system [4,6].

As for conventional energy trapping devices [6], the forced modes are found as the linear combinations of the eigen-modes at V=0 that verify the complete equations. After going back to the usual displacement and potential, the electrical parameters characterizing the resonators are also determined in a similar manner as for conventional energy trapping devices [6].

It should be noticed that “inverse” energy trapping devices using less symmetrical geometries can also be modelized using methods derived from those already presented in reference [4 and 6] for conventional energy trapping resonators. In this case, the displacement can be represented as a linear combination of all the eigen modes existing for a circular electrode For the electroded (central) region of a resonator with a non circular electrode and an external region, we have now:

$$\hat{u}_{1e}^{nm} = \sum_m B_m^n J_m(\hat{k}^n.r)\cos mt.\sin(n\pi x_2/2h)$$

The solution of the problem is then found using a discretization of the continuity and boundary conditions (expressing them at q points of the limits of regions) and a truncation of the previous infinite sums [6].

The comparison of the expressions of the displacement and of the properties of the less symmetrical resonators with those that are obtained as above for resonators with a revolution symmetry, ***shows the optimal character of the resonator geometries possessing a revolution symmetry. They lead to a reduced number of excited modes at equal surface and to better electrical properties at equal surface, etc..).*** *These facts are the consequence of the properties observed and demonstrated in a more general case, in reference [5 and 6].*

III. Thin Film Resonators with circular or annular electrodes using the extensional modes.

Only the modes for m=0 (radial modes) are electrically excited with plain electrodes of revolution geometry (in well realized resonators, the current and the admittance vanish for the modes having one or several diametric symmetries (for m>0)).

The computed eigen mode shapes of the 4 first modes (among 5 existing) of a resonator made with a film of AlN (2h=3µm) and large circular electrodes (diameter=100µm, R=-1%) are represented in figures 3 and 4. We can observe that the amplitude of vibration decrease fastly outside the electrodes for all the eigen modes. Two modes corresponding to m=0 are electrically strongly excited (mode 1 and 3) so that such a resonator can not be used in a filter.

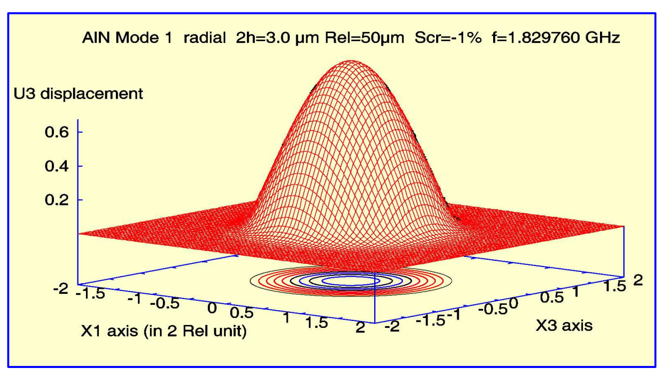

Figure 3: Computed mode shape of the fundamental extensional mode.

To obtain filter type resonators it is necessary to chose the electrode dimensions and R (R’, R’’) so to have only 1 radial mode [5]. This is made by numerical calculations in varying the parameters (electrode diameter, R, etc….). Such calculations were made for resonators using an AlN film 3µm thick. Many solutions are found using various groove depth and various metal and/or dielectric film thickness.

Two examples of design are considered in table 1. We observe that for small and medium “apparent” mass loading (R<0), the maximal electrode diameter giving a single mode response is of the order of 13 to 17 times the film thickness, depending on the other parameters. We shall recall that for UHF energy trapping resonators it nearly always preferable to use electrodes made with high electrical conductivity and small density metal films having a small (but sufficient) thickness.
The two examples concerning the properties of resonators with circular electrodes and a single mode response (only one radial eigen mode) that are presented in table 1 were computed for AlN films having a thickness (2h) equal to 3.0 micrometers. Depending on the apparent mass loading (-R in the table) and the exact geometry of the resonators (dimensions of the electrodes, R’, R”), the resonance frequencies of the filter type resonators, we have computed were situated in the range 1840-1865 MHz.

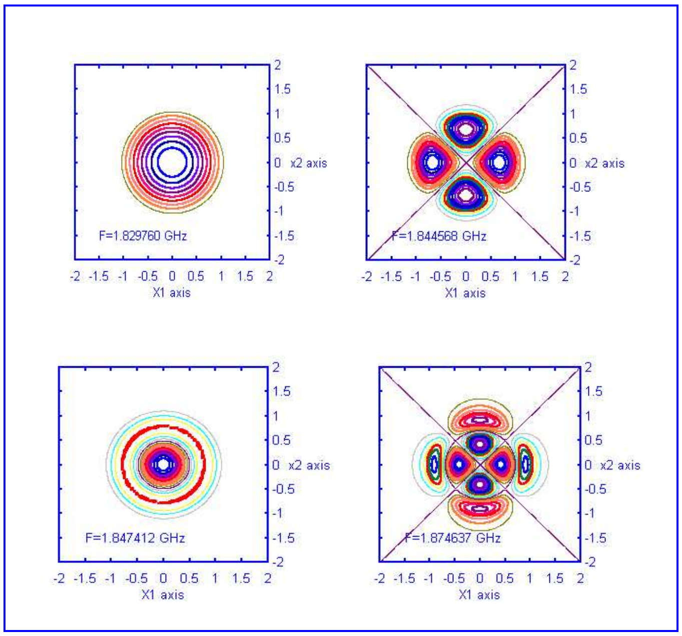

Figure 4: The four first eigen modes of a resonator with a large circular electrodes

In table 1 we give also computed examples of resonators having annular electrodes. It should be noticed that the electrodes diameters given in the table for the resonators with circular electrodes are close to the maximal ones that give a single mode response; **whereas single mode responses can be obtained with nearly as large dimensions as wanted for the resonators with annular electrodes.** The condition to obtain a single mode response is then (in a first approximation), that the difference between the outer and the inner diameters of the electrode are lower than a value close to the maximal diameter of a resonator with circular electrodes giving a single mode response (with the same R and 2h).

Table 1: Filter type resonators (circular or annular electrodes with quite low “apparent” mass loading).

	R (%)	Electrode Diam.	Ext. electrode diameter	Int. electrode diameter	Inductance	P max
1	-0.5%	40µm	N.A.	N.A.	L=4584 µH	1
2	-0.3%	50 µm	N.A.	N.A.	L=2894 µH	1
3	-1%	NA	150 µm	115 µm	L=820 nH	1
4	-0.5%	NA	230 µm	190 µm	L=442 nH	1

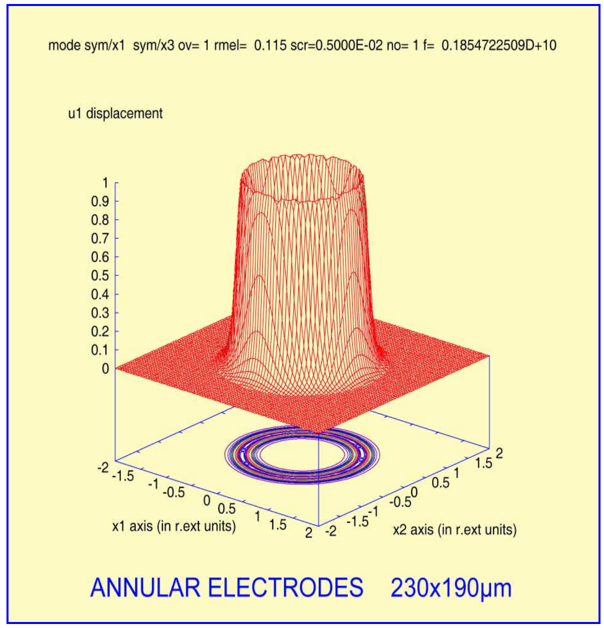

Figure 5: Mode shape of the unique mode of a resonator with an annular electrode. (resonator 4 of table 1, n=1,m=0, p=1).

IV. Application to UHF Filtering.

The extensional mode resonators using circular electrodes can be used to make miniaturized reception filters for UHF communication systems using such small resonators. This can be done using several circuit topologies, but we have shown that a very efficient technique with this material which has only a moderate coupling coefficient is to use [5] full symmetrical lattice filters “enlarged” with serial inductances. Such circuits can use resonators with quite large impedance levels (larges inductances) to obtain filters with low termination impedances.
The response given in figure 6 was computed for such a circuit using a full lattice with 2 resonator per arms and 4 serial inductances (extracted from the lattice and situated at the input and output ports) in choosing termination

impedances of 15 Ohms. The 4 different kinds of resonators (on 8) used in the filter have all inductances greater or equal to that of resonator 2 of table 1. The minimal value of inductance chosen being exactly 2894µH, two resonators use this value, two other have inductances which are only very slightly higher and the four other ones have inductances slightly larger than those of resonator 1 of table 1.

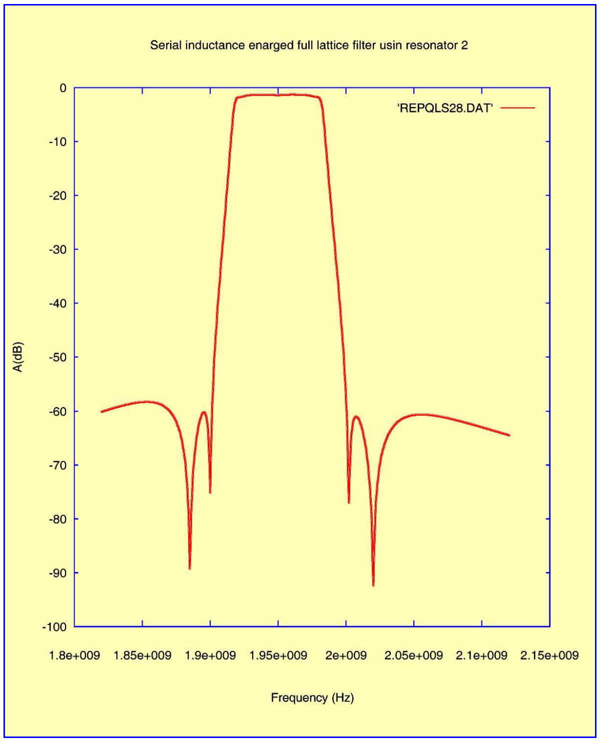

Figure 6: Computed response of a miniaturized lattice filter using 8 resonators with circular electrodes.

The resonators using annular electrodes can have much lower inductances (larger surfaces) they can transmit a quite large power. Such resonators can be advantageously used in full lattice filter enlarged with parallel inductances which have important advantages over most other filter topologies concerning the power transmitted and the performances of the responses achievable with moderate coupling materials. Some details about the possibility and the interest of these circuits as compared to more conventional ones can be found in [5]. In this reference, more general designs of devices (monolithic filters etc…) using similar or derived geometries are also considered.

V. CONCLUSION

A simple theory of devices using the inverse energy trapping of the extensional essentially thickness modes was presented. It consider particularly the case of the thickness extensional mode existing in piezoelectric plates with transverse isotropy (having at least a three fold axis of symmetry normal to the plate). This theory is particularly adapted to the case of miniaturized resonators and other MEMS made using textured or polarized piezoelectric thin film with 6mm symmetry. The possibility to obtain resonators with a large variety of equivalent schemes was demonstrated. The main interest of energy trapping resonators is to have less unwanted modes (no anharmonic modes and much less coupling with plate modes at the edges of the plate), to be less sensitive to their mechanical environment, to have larger Q factors (reduced losses at their fixations) and to have coupling coefficients close to the maximal value possible with the material used. Thin electrodes can most often be used in their design which leads to more important advantages concerning most of the previous points.

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